# Inverse Trigonometric Functions

## Question1

**If**
$\theta = \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \tan^{-1}\left(\frac{1}{31}\right)$
**, then** $\tan\theta =$

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**Options:**

A.

$\frac{3}{5}$

B.

1

C.

$\frac{5}{7}$

D.

$\frac{7}{9}$

**Answer: C**

## Solution:

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \tan^{-1}\left(\frac{1}{31}\right) \\ &= \sum_{k=1}^{5} \tan^{-1}\left(\frac{1}{k^2+k+1}\right)\end{aligned}$$

$$and\ \tan^{-1}(k+1)-\tan^{-1}(k)=\tan^{-1}\left(\frac{1}{k^2+k+1}\right)$$

$$\begin{aligned}
&\therefore \theta=\sum_{k=1}^{5}\left(\tan^{-1}(k+1)-\tan^{-1}(k)\right)\\
&=\tan^{-1}(2)-\tan^{-1}(1)+\tan^{-1}(3)\\
&-\tan^{-1}(2)+\tan^{-1}(4)-\tan^{-1}(3)\\
&+\tan^{-1}(5)-\tan^{-1}(4)+\tan^{-1}(6)\\
&-\tan^{-1}(5)\\
&=\tan^{-1}(6)-\tan^{-1}(1)\\
&=\tan^{-1}\left(\frac{6-1}{1+6\times 1}\right)=\tan^{-1}\left(\frac{5}{7}\right)\\
&\Rightarrow \tan\theta=\frac{5}{7}
\end{aligned}$$

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# Question2

**If $\tan^{-1}x=\cot h^{-1}y=\log\sqrt{5}$, then $\tan^{-1}(xy)=$**

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**Options:**

A.

$\frac{\pi}{4}$

B.

$\frac{\pi}{3}$

C.

$\frac{\pi}{6}$

D.

$\frac{3\pi}{4}$

**Answer: A**

## Solution:

$\because \coth^{-1} y = \tanh^{-1}\left(\frac{1}{y}\right)$

$$\text{so, } \tanh^{-1} x = \tanh^{-1}\left(\frac{1}{y}\right)$$
$$\Rightarrow \quad x = \frac{1}{y}$$
$$\Rightarrow \quad xy = 1$$

Therefore, $\tan^{-1}(xy) = \tan^{-1}(1) = \frac{\pi}{4}$

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# Question3

**If $f(x) = 2 + \left|\sin^{-1} x\right|$ and $A = \{x \in R/f^1(x)$ exists $\}$, then $A =$**

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**Options:**

A.

$\{0\}$

B.

$[-1, 1]$

C.

$(-\infty, -1) \cup (1, \infty)$

D.

$(-1, 0) \cup (0, 1)$

**Answer: D**

## Solution:

$\because f(x) = 2 + \left|\sin^{-1} x\right|$

Domain of $\sin^{-1} x$ is $x \in [-1, 1]$

So, the domain of $f(x)$ is $[-1, 1]$

and $\left|\sin^{-1} x\right|$ is not differentiable at $x = 0$

So, the set A where $f'(x)$ exists is

$(-1, 1) - \{0\}$

Hence, $f(x)$ is differentiable at all points in $(-1, 0) \cup (0, 1)$

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# Question4

$$\tan\left(2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)\right) =$$

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**Options:**

A.

$\frac{1}{\sqrt{3}}$

B.

$\sqrt{3}$

C.

1

D.

$3/7$

**Answer: C**

## Solution:

We have,

$$\tan\left(2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}\right)$$

$$=\tan\left(\tan^{-1}\left(\frac{2/3}{1-1/9}\right)+\tan^{-1}\frac{1}{7}\right)\left\{\because 2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}\right\}$$

$$=\tan\left[\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{1}{7}\right)\right]$$

$$=\tan\left[\tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\cdot\frac{1}{7}}\right)\right]\left\{\begin{array}{l}\because \tan^{-1}x+\tan^{-1}y\\ =\tan^{-1}\left(\frac{x+y}{1-xy}\right)\end{array}\right\}$$

$$=\tan\left(\tan^{-1}\left(\frac{25}{25}\right)\right)=1$$

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# Question5

$$\tanh^{-1}\left(\frac{1}{3}\right)+\coth^{-1}(3)=$$

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**Options:**

A.

$\operatorname{sech}^{-1}\left(\frac{1}{3}\right)$

B.

$\operatorname{cosech}^{-1}\left(\frac{1}{3}\right)$

C.

$\cosh^{-1}\left(\frac{4}{3}\right)$

D.

$\sinh^{-1}\left(\frac{3}{4}\right)$

**Answer: D**

## Solution:

We have,

$$\tanh^{-1}\left(\frac{1}{3}\right)+\coth^{-1}(3)$$
$$=\tanh^{-1}\left(\frac{1}{3}\right)+\tanh^{-1}\left(\frac{1}{3}\right)$$
$$=2\tanh^{-1}\left(\frac{1}{3}\right)$$
$$=\sinh^{-1}\left(\frac{2\times\frac{1}{3}}{1-\frac{1}{9}}\right)=\sinh^{-1}\left(\frac{2/3}{8/9}\right)$$
$$\left\{\because 2\tanh^{-1}x=\sinh^{-1}\left(\frac{2x}{1-x^2}\right)\right\}$$
$$=\sinh^{-1}\left(\frac{3}{4}\right)$$

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# Question6

**If $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $\left(\frac{d^2y}{dx^2}\right)_{x=2}=k$, then $25k=$**

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**Options:**

A.

$(-3)^2$

B.

$(-2)^3$

C.

3

D.

$(-2)^5$

**Answer: B**

## Solution:

Given, $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$\because \quad \sin^{-1}\frac{2x}{1+x^2}=2\tan^{-1}x$$

$$\therefore \quad y=2\tan^{-1}x \Rightarrow \frac{dy}{dx}=\frac{2}{1+x^2}$$

$$\Rightarrow \frac{d^2y}{dx^2}=\frac{(-2)2x}{(1+x^2)^2}=-\frac{4x}{(1+x^2)^2}$$

$$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=2}=-\frac{8}{25}=k$$

$$\therefore 25k=-8=(-2)^3$$

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# Question7

**If $f(x)=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ and $g(x)=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then the derivative of $f(x)$ with respect to $g(x)$ is**

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**Options:**

A.

$\frac{1+x^2}{4\sqrt{1-x^2}}$

B.

$\frac{(1-x^2)}{4\sqrt{1+x^2}}$

C.

$-\frac{4(1-x^2)}{\sqrt{1+x^2}}$

D.

$-\frac{4(1+x^2)}{\sqrt{1-x^2}}$

**Answer: D**

## Solution:

Given,

$$f(x)=\sec^{-1}\left(\frac{1}{2x^2-1}\right), g(x)=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$

put, $x=\cos\theta$ $\quad$ $x=\tan\phi$

$f(x)=\sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$ $\quad$ $g(x)=\tan^{-1}\left(\frac{\sec\phi-1}{\tan\phi}\right)$

$$=\sec^{-1}(\sec 2\theta)$$
$$=2\theta$$
$$=2\cos^{-1}x$$
$$\Rightarrow f'(x)=\frac{-2}{\sqrt{1-x^2}}$$

$$=\tan^{-1}\left(\frac{1-\cos\phi}{\sin\phi}\right)$$
$$=\tan^{-1}\left(\frac{2\sin^2\phi/2}{2\sin\phi/2\cos\phi/2}\right)$$
$$=\tan^{-1}(\tan\phi/2)$$
$$=\phi/2$$
$$=\frac{1}{2}\tan^{-1}x$$
$$\Rightarrow g'(x)=\frac{1}{2\left(1+x^2\right)}$$

Differentiating $f(x)$ w.r.t $g(x)$ is

$$=\frac{f'(x)}{g'(x)}=\frac{-4\left(1+x^2\right)}{\sqrt{1-x^2}}$$

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# Question8

**If $A=\left\{x\in R/\sin^{-1}\left(\sqrt{x^2+x+1}\right)\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right\}$ and $B=\left\{y\in R/y=\sin^{-1}\left(\sqrt{x^2+x+1}\right),x\in A\right\}$, then**

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**Options:**

A.

$A\cap B\neq\phi$

B.

$A\cap B^C=[0,1]$

C.

$A^C \cap B = \left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

D.

$A \cup B = R - \left\{[-1, 0] \cup \left[\frac{\pi}{3}, \frac{\pi}{2}\right]\right\}$

**Answer: C**

## Solution:

$A = \left\{x \in R / \sin^{-1}\left(\sqrt{x^2+x+1}\right) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right\}$

and

$B = \left\{y \in R / y = \sin^{-1}\left(\sqrt{x^2+x+1}\right), x \in A\right\}$

Since, domain of $\sin^{-1}(x)$ is $[-1, 1]$

So, domain of $\sin^{-1}\sqrt{x^2+x+1}$ is real, so

so

$$\begin{aligned} & \sqrt{x^2+x+1} \in [-1, 1] \\ \Rightarrow \quad & \sqrt{x^2+x+1} \in [0, 1] \end{aligned}$$

( $\because$ square root is always non-negative)

$$\begin{aligned} \Rightarrow \quad & x^2+x+1 \leq 1 \\ \Rightarrow \quad & x^2+x \leq 0 \\ \Rightarrow \quad & x(x+1) \leq 0 \end{aligned}$$

So, $x \in [-1, 0]$ for set $A$

Thus, $A = [-1, 0]$

Now, for set $B$, since $x \in A = [-1, 0]$, the range values for $\sqrt{x^2+x+1}$ for $x \in [-1, 0]$ we will find.

$$\begin{aligned} \text{At } x = -1, & \sqrt{(-1)^2 - 1 + 1} \\ & = \sqrt{1 - 1 + 1} = 1 \\ \text{At } x = 0, & \sqrt{0^2 + 0 + 1} = \sqrt{0+1} = 1 \\ \text{At } x = -\frac{1}{2}, & \sqrt{\left(\frac{-1}{2}\right)^2 - \frac{1}{2} + 1} \\ & = \sqrt{\frac{1}{4} - \frac{1}{2} + 1} \\ & = \sqrt{\frac{3}{4}} \end{aligned}$$

So, $\sqrt{x^2+x+1} \in \left[\sqrt{\frac{3}{4}}, 1\right] = \left[\frac{\sqrt{3}}{2}, 1\right]$

Thus, $y = \sin^{-1}\left(\sqrt{x^2+x+1}\right)$

$$\in \left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right), \sin^{-1}(1)\right]$$
$$= \left[\frac{\pi}{3}, \frac{\pi}{2}\right]$$

Hence, $B = \left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

$\Rightarrow$ Now, $A \cap B = [-1, 0] \cap \left[\frac{\pi}{3}, \frac{\pi}{2}\right] = \phi$

So, $A \cap B \neq \phi$ is false.

$\Rightarrow A \cap B^C = [-1, 0] \cap \left[\frac{\pi}{3}, \frac{\pi}{2}\right]^C$ and the intersection with any set outside this cannot be $[0, 1]$,

So, $A \cap B^C = [0, 1]$ is false.

$\Rightarrow A^C$ is everything except $[-1, 0]$.

Since $B = (\pi/3, \pi/2) \subset R/(-1, 0)$

$\Rightarrow \quad A^C \cap B$ is true.

$\Rightarrow \quad A \cup B = R - \left\{[-1, 0] \cup \left[\frac{\pi}{3}, \frac{\pi}{2}\right]\right\}$

So, union of $A$ and $B$ is the complement of their own union.

That's impossible.

Thus, $A^C \cap B = \left[\frac{\pi}{3}, \frac{\pi}{2}\right]$, option (c) is correct.

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# Question9

## The domain of the function, $f(x) = \sqrt{\log_e\left(\frac{1}{x^2-4x+4}\right)} + \sin^{-1}\left(x^2-2\right)$ is

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**Options:**

A.

$[1, 3]$

B.

$[1,3)$

C.

$[1,\sqrt{3}]$

D.

$[1,\sqrt{3})$

**Answer: C**

## Solution:

Given function,

$$f(x)=\sqrt{\log_e\left(\frac{1}{x^2-4x+4}\right)}+\sin^{-1}\left(x^2-2\right)$$

Since, log is defined for positive values only.

So, $\frac{1}{x^2-4x+4}>0$

Also since, $x^2-4x+4=(x-2)^2\geq 0$ this holds for $x\neq 2$

$\Rightarrow\quad \ln\left(\frac{1}{(x-2)^2}\right)\geq 0$

Now, we know that $\ln(t)$ is strictly increasing for $t>0$ and $\ln(1)=0$, the inequality

$\ln\frac{1}{(x-2)^2}\geq 0$ is equivalent to

$\frac{1}{(x-2)^2}\geq 1\Leftrightarrow(x-2)^2\leq 1$

$\Rightarrow\quad |x-2|\leq 1$

i.e., $\quad 1\leq x\leq 3$

But log is undefined for $x=2$

$\therefore x\in[1,3]-\{2\}$

For $\sin^{-1}\left(x^2-2\right)$ to be real, $x^2-2\in[-1,1]$

$$\Rightarrow\quad -1\leq x^2-2\leq 1$$
$$\Rightarrow\quad 1\leq x^2\leq 3\Rightarrow 1\leq|x|\leq\sqrt{3}$$
$$\Rightarrow\quad x\in[-\sqrt{3},-1]\cup[1,\sqrt{3}]\Rightarrow x\in[1,\sqrt{3}]$$

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# Question10

**If** $\cot\left(\cos^{-1} x\right) = \sec\left\{\tan^{-1}\left(\frac{a}{\sqrt{b^2-a^2}}\right)\right\}, b > a$ **then** $x =$

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**Options:**

A.

$\frac{b}{\sqrt{2b^2-a^2}}$

B.

$\frac{a}{\sqrt{2b^2-a^2}}$

C.

$\frac{\sqrt{b^2-a^2}}{a}$

D.

$\frac{\sqrt{b^2-a^2}}{b}$

**Answer: A**

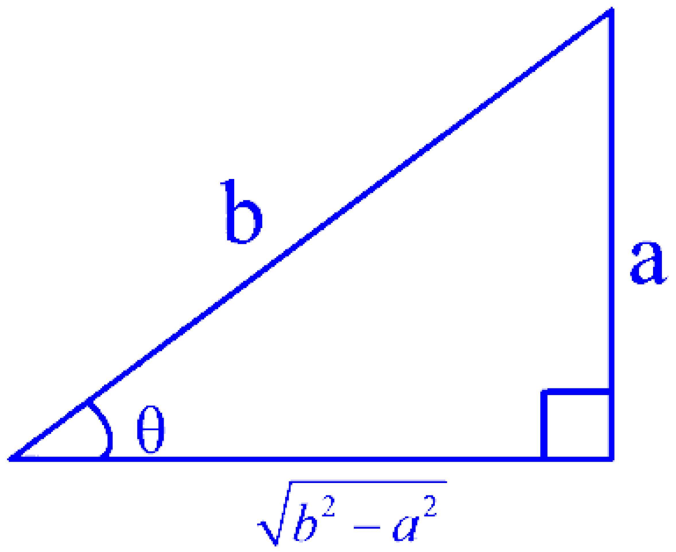
## Solution:

Given,

$\cot\left(\cos^{-1} x\right) = \sec\left\{\tan^{-1}\left(\frac{a}{\sqrt{b^2-a^2}}\right)\right\}, b > a$

Let $\tan^{-1}\frac{a}{\sqrt{b^2-a^2}} = \theta \Rightarrow \tan\theta = \frac{a}{\sqrt{b^2-a^2}}$

$\sec\theta = \frac{b}{\sqrt{b^2-a^2}}$



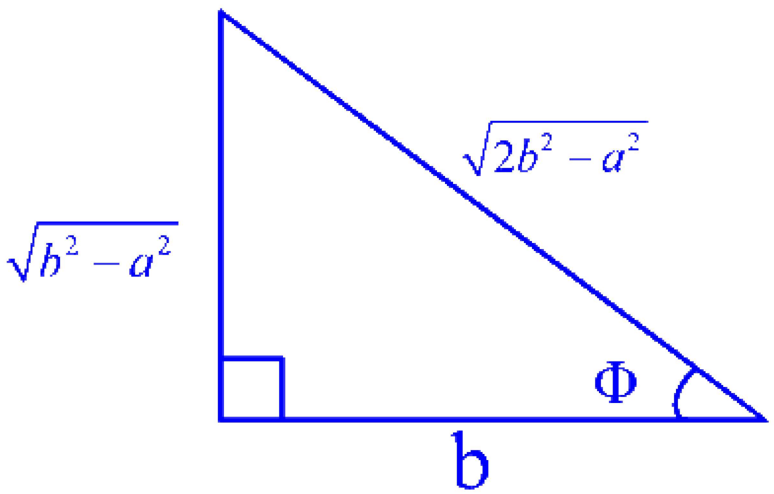

$$\therefore \cot\left(\cos^{-1} x\right) = \sec\left\{\tan^{-1} \frac{a}{\sqrt{b^2 - a^2}}\right\}$$

$$\sec\theta = \frac{b}{\sqrt{b^2 - a^2}}$$

$$\Rightarrow \quad \cos^{-1} x = \cot^{-1}\left(\frac{b}{\sqrt{b^2 - a^2}}\right)$$

Again, let $\cot^{-1}\left(\frac{b}{\sqrt{b^2 - a^2}}\right) = \phi$

$$\Rightarrow \quad \cot\phi = \frac{b}{\sqrt{b^2 - a^2}}$$




$$\Rightarrow \quad \cos\phi = \frac{b}{\sqrt{b^2 - a^2}}$$

Now, $\cos^{-1} x = \phi$

$$\Rightarrow \quad x = \cos\phi = \frac{b}{\sqrt{2b^2 - a^2}}$$

$$\therefore \quad x = \frac{b}{\sqrt{2b^2 - a^2}}$$

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# Question11

**If** $\sinh^{-1} x = \log 3$ **and** $\cosh^{-1} y = \log \frac{3}{2}$**, then** $\tanh^{-1}(x - y) =$

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**Options:**

A.

$\log \sqrt{\frac{5}{3}}$

B.

$\log \frac{5}{3}$

C.

$\log \frac{4}{3}$

D.

$\log \frac{2}{\sqrt{3}}$

**Answer: A**

## Solution:

$$\begin{aligned}
\text{Given, } \sin h^{-1} x &= \log 3 \\
\cos h^{-1} y &= \log \frac{3}{2} \\
\because \sin h^{-1} x &= \log(3) \\
\Rightarrow \quad x &= \sin h(\log 3) \\
&= \frac{e^{\log 3} - e^{-\log 3}}{2} \\
&= \frac{3 - \frac{1}{3}}{2} = \frac{8}{6} = \frac{4}{3}
\end{aligned}$$

$$\text{And } \cos h^{-1} y = \log\left(\frac{3}{2}\right)$$

$$\Rightarrow \quad y = \cosh\left(\log \frac{3}{2}\right)$$

$$\Rightarrow \quad \frac{e^{\log\left(\frac{3}{2}\right)}+e^{-\log\left(\frac{3}{2}\right)}}{2}=\frac{\frac{3}{2}+\frac{2}{3}}{2}=\frac{13}{12}$$

$$x-y=\frac{4}{3}-\frac{13}{12}$$

$$=\frac{16-13}{12}=\frac{3}{12}=\frac{1}{4}$$

Now, we know that

$\tan h^{-1}(z)=\frac{1}{2}\ln\left(\frac{1+z}{1-z}\right)$

Now, $z=x-y$

So, $\tan h^{-1}(x-y)=\frac{1}{2}\ln\left(\frac{1+x-y}{1-(x-y)}\right)$

$$=\frac{1}{2}\ln\left(\frac{1+x-y}{1-x+y}\right)$$

$$=\frac{1}{2}\ln\left(\frac{1+\frac{1}{4}}{1-\frac{1}{4}}\right)=\frac{1}{2}\ln\left(\frac{5/4}{3/4}\right)$$

$$=\frac{1}{2}\ln\left(\frac{5}{3}\right)=\ln\left(\sqrt{\frac{5}{3}}\right)$$

So, $\tan h^{-1}(x-y)=\ln\left(\sqrt{\frac{5}{3}}\right)$

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# Question12

**The number of solution of $\tan^{-1}1+\frac{1}{2}\cos^{-1}x^2-\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=0$ is**

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**Options:**

A.

3

B.

0

C.

1

D.

infinitely many

**Answer: D**

## Solution:

$$\tan^{-1}(1)+\frac{1}{2}\cos^{-1}\left(x^2\right)$$
$$-\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=0$$
$$\because \tan^{-1}(1)=\pi/4 \text{ and let } \theta=\cos^{-1}\left(x^2\right)$$
$$\Rightarrow x^2=\cos\theta$$
$$\text{and } 1+x^2=1+\cos x, 1-x^2=1-\cos x$$
$$=\frac{\pi}{4}+\frac{\theta}{2}-\tan^{-1}\left(\frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}\right)$$
$$=\frac{\pi}{4}+\frac{\theta}{2}-\tan^{-1}\left(\frac{\sqrt{2}\cos\theta/2+\sqrt{2}\sin\theta/2}{\sqrt{2}\cos\theta/2-\sqrt{2}\sin\theta/2}\right)$$
$$=\frac{\pi}{4}+\frac{\theta}{2}-\tan^{-1}\left(\frac{\cos\theta/2+\sin\theta/2}{\cos\theta/2-\sin\theta/2}\right)$$
$$\because \frac{\cos x+\sin x}{\cos x-\sin x}=\tan\left(\frac{\pi}{4}+x\right)$$
$$=\frac{\pi}{4}+\frac{\theta}{2}-\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right)$$
$$=\frac{\pi}{4}+\frac{\theta}{2}-\frac{\pi}{4}-\frac{\theta}{2}=0$$

Hence, the identify is true for all $x$ Infinite many solutions.

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# Question13

$$\tanh^{-1}(\sin\theta)=$$

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**Options:**

A.

$\sinh^{-1}(\operatorname{cosec}\theta)$

B.

$\sinh^{-1}(\sec\theta)$

C.

$\cosh^{-1}(\operatorname{cosec}\theta)$

D.

$\cosh^{-1}(\sec\theta)$

**Answer: D**

## Solution:

$$\because \tanh^{-1}(x)=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)(\text{ for }|x|<1)$$

$$\text{so, } \tanh^{-1}(\sin\theta)=\frac{1}{2}\ln\left(\frac{1+\sin\theta}{1-\sin\theta}\right)$$

$$\frac{1+\sin\theta}{1-\sin\theta}=\frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}=\left(\frac{1+\sin\theta}{\cos\theta}\right)^2$$

$$=\frac{1}{2}\ln\left(\frac{1+\sin\theta}{\cos\theta}\right)^2$$

$$=\ln\left(\frac{1+\sin\theta}{\cos\theta}\right)\qquad\ldots(i)$$

$$\text{and } \cosh^{-1}(x)=\ln\left(x+\sqrt{x^2-1}\right)$$

$$\therefore \cosh^{-1}(\sec\theta)=\ln\left(\frac{1}{\cos\theta}+\sqrt{\frac{1-\cos^2\theta}{\cos^2\theta}}\right)$$

$$=\ln\left(\frac{1+\sin\theta}{\cos\theta}\right)\qquad\ldots(ii)$$

By comparing Eqs. (i) and (ii), we get
$\tanh^{-1}(\sin\theta)=\cosh^{-1}(\sec\theta)$

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# Question14

**The interval in which the function $f(x)=\tan^{-1}(\sin x+\cos x)$ is an increasing function is**

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**Options:**

A.

$\left(0, \frac{\pi}{2}\right)$

B.

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

C.

$\left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$

D.

$\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

**Answer: C**

## Solution:

$$\because \sin x + \cos x = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right)$$
$$\therefore f(x) = \tan^{-1}\left(\sqrt{2} \sin\left(x + \frac{\pi}{4}\right)\right)$$
$$\text{Now, } f'(x) = \frac{1}{1 + \left(\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)\right)^2}$$
$$\left(\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)\right)$$

$= \frac{\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)}{1+2\sin^2\left(x+\frac{\pi}{4}\right)} \rightarrow$ always $> 0$

for increasing $\sqrt{2}\cos\left(x + \frac{\pi}{4}\right) > 0$

$\Rightarrow \quad \cos\left(x + \frac{\pi}{4}\right) > 0$

So, $\quad \frac{-\pi}{2} + 2k\pi < x + \frac{\pi}{4} < \frac{\pi}{2} + 2k\pi$

$\Rightarrow \quad \frac{-3\pi}{4} + 2k\pi < x < \frac{\pi}{4} + 2k\pi$

for the principal interval,

let $k = 0$

Thus, the interval is

$x \in \left(\frac{-3\pi}{4}, \frac{\pi}{4}\right)$

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# Question15

**The range of the real valued function $f(x) = \cos^{-1}\left(\frac{3}{\sqrt{9x^2-12x+22}}\right)$ is**

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**Options:**

A.

$\left(0, \frac{\pi}{4}\right]$

B.

$\left[\frac{\pi}{4}, \frac{\pi}{2}\right)$

C.

$[0, \pi]$

D.

$\left[\frac{\pi}{6}, \frac{\pi}{2}\right)$

**Answer: B**

## Solution:

Given, $f(x) = \cos^{-1}\left(\frac{3}{\sqrt{9x^2-12x+22}}\right)$

Since, $9x^2 - 12x + 22 = 9$

$$[\underbrace{\left(x - \frac{2}{3}\right)^2 + 2]}_{\geq 2} \geq 18$$

$$\Rightarrow \quad \sqrt{9\left(x - \frac{3}{2}\right)^2 + 18} \geq \sqrt{18}$$

$$\Rightarrow \quad 0 < \frac{1}{\sqrt{9x^2-12x+22}} \leq \frac{1}{\sqrt{18}}$$

$$\Rightarrow \quad 0 < \frac{3}{\sqrt{9x^2-12x+22}} \leq \frac{3}{\sqrt{18}}$$

$$\Rightarrow \quad 0 < \frac{3}{\sqrt{9x^2-12x+22}} \leq \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \cos^{-1} 0 > \cos^{-1}\left(\frac{3}{\sqrt{9x^2 - 12x + 22}}\right)$$
$$\geq \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$
$$\Rightarrow \quad \frac{\pi}{4} \leq f(x) < \frac{\pi}{2}$$

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# Question16

**If the equation $2\cot^{-1}\left(x^2 + 2x + k\right) = \pi - 3\tan^{-1}\left(x^2 + 2x + k\right)$ has two distinct real solutions, then all the values of $k$ lie in the interval**

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**Options:**

A.

$(-1, 2)$

B.

$(1, \infty)$

C.

$(-\infty, \infty)$

D.

$(-\infty, 1)$

**Answer: D**

## Solution:

As we know that,

$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$

So, given equation reduces to

$$2\left(\frac{\pi}{2}-\tan^{-1}\left(x^2+2x+k\right)\right)$$
$$=\pi-3\tan^{-1}\left(x^2+2x+k\right)$$
$$\Rightarrow \quad \tan^{-1}\left(x^2+2x+k\right)=0$$
$$\Rightarrow \quad x^2+2x+k=0$$

Given Eq. (i) has two distinct real solution. So, its discriminant value is always greater than zero.

So, (4) $-4k>0$

$\Rightarrow \quad k<1 \Rightarrow k \in(-\infty, 1)$

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# Question17

$\sec h^{-1}(\sin \alpha)=$

## AP EAPCET 2025 - 23rd May Morning Shift

**Options:**

A.

$\log \left(\sin \alpha+\sqrt{\sin^2 \alpha-1}\right)$

B.

$\log(\tan \alpha+1)$

C.

$\log \left(\cot \frac{\alpha}{2}\right)$

D.

$\log \left(\frac{1+\tan \alpha}{2 \sin \alpha}\right)$

**Answer: C**

## Solution:

$$\because \operatorname{sech}^{-1}(x) = \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right)$$

$$\therefore \operatorname{sech}^{-1}(\sin\alpha) = \ln\left(\frac{1}{\sin\alpha} + \sqrt{\frac{1}{\sin^2\alpha} - 1}\right)$$

$$= \ln\left(\frac{1}{\sin\alpha} + \sqrt{\frac{1 - \sin^2\alpha}{\sin^2\alpha}}\right)$$

$$= \ln\left(\frac{1}{\sin\alpha} + \frac{\cos\alpha}{\sin\alpha}\right)$$

$$= \ln\left(\frac{1 + \cos\alpha}{\sin\alpha}\right) = \ln\left(\frac{2\cos^2\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}\right)$$

$$= \ln\left(\cot\frac{\alpha}{2}\right)$$

----------------------------------------------------------------------------------------------------

# Question18

**If $y = \log\left(\sec\left(\tan^{-1}x\right)\right)(x > 0)$, then $\frac{dy}{dx}$ at $x = 1$ is**

## AP EAPCET 2025 - 23rd May Morning Shift

**Options:**

A.

1

B.

3

C.

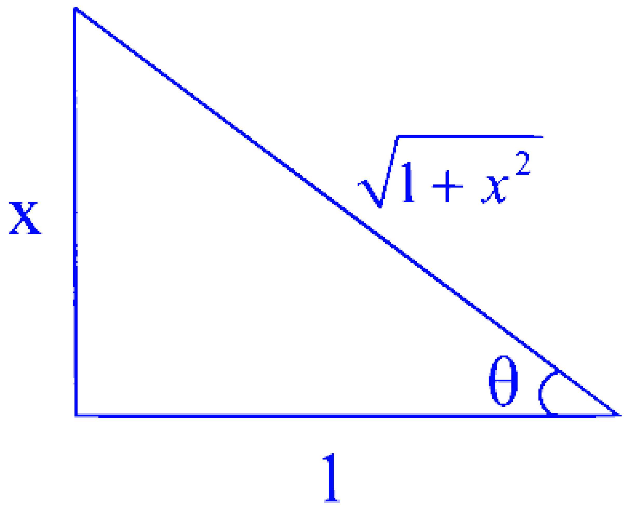
$\frac{1}{2}$

D.

$\frac{3}{2}$

**Answer: C**

## Solution:




Given $y=\log\left(\sec\left(\tan^{-1}x\right)\right), x>0$

$$=\log\left(\sec\left(\sec^{-1}\frac{\sqrt{1+x^2}}{1}\right)\right)$$

$$y=\log\left(\sqrt{1+x^2}\right)$$

$\therefore$ On differentiate w.r.t. $x$,

$$\frac{dy}{dx}=\frac{1}{\sqrt{1+x^2}}\times\frac{1}{2\sqrt{1+x^2}}\times(2x)=\frac{x}{\left(1+x^2\right)^2}$$

$$\therefore \left.\frac{dy}{dx}\right|_{x=1}=\frac{1}{1+(1)^2}=\frac{1}{2}$$

---

# Question19

If $y=\mathrm{Sin}^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$ and $\frac{-3\pi}{2}<x<\frac{-\pi}{2}$, then $\frac{dy}{dx}=$

## AP EAPCET 2025 - 23rd May Morning Shift

**Options:**

A.

$$-\frac{\left|\operatorname{cosec}\frac{x}{2}\right|}{2\sqrt{\sin^2\frac{x}{2}-\cos^2\frac{x}{2}}}$$

B.

$\frac{\left|\sec \frac{x}{2}\right|}{2 \sqrt{\cos x}}$

C.

$\frac{\cos \frac{x}{2}}{2 \sqrt{\cos x}}$

D.

$\frac{\cos \frac{x}{2}}{\sqrt{\cos x}}$

**Answer: B**

## Solution:

Given,

$$y=\sin ^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$$

$$\Rightarrow y=\sin ^{-1}\left(\frac{(\sqrt{1+\sin x}+\sqrt{1-\sin x})^{2}}{(\sqrt{1+\sin x})^{2}-(\sqrt{1-\sin x})^{2}}\right)$$

$$=\sin ^{-1}\left(\frac{1+\sin x+1-\sin x+2 \sqrt{1-\sin ^{2} x}}{1+\sin x-1+\sin x}\right)$$

$$=\sin ^{-1}\left(\frac{2+2|\cos x|}{2 \sin x}\right)=\sin ^{-1}\left(\frac{1-\cos x}{\sin x}\right)\left(\begin{array}{l}\because \frac{-3 \pi}{2}<x<\frac{-\pi}{2} \\ \therefore|\cos x|=-\cos x\end{array}\right)$$

$$=\sin ^{-1}\left(\frac{2 \sin ^{2}\left(\frac{x}{2}\right)}{2 \sin \left(\frac{x}{2}\right) \cos \left(\frac{x}{2}\right)}\right)=\sin ^{-1}\left(\tan \frac{x}{2}\right)$$

$$\therefore \frac{d y}{d x}=\frac{1}{\sqrt{1-\tan ^{2}\left(\frac{x}{2}\right)}} \times \sec ^{2}\left(\frac{x}{2}\right) \times \frac{1}{2}$$

$$=\frac{\left|\cos \frac{x}{2}\right|}{\sqrt{\cos ^{2} \frac{x}{2}-\sin ^{2} \frac{x}{2}}} \times \frac{1}{\left|\cos \frac{x}{2}\right|^{2}} \times \frac{1}{2}=\frac{\left|\sec \frac{x}{2}\right|}{2 \sqrt{\cos x}}$$

---

# Question20

**If $\frac{1}{2} \sin ^{-1}\left(\frac{3 \sin 2 \theta}{5+4 \cos 2 \theta}\right)=\tan ^{-1} x$, then $x=$**

## AP EAPCET 2025 - 22nd May Evening Shift

**Options:**

A.

$\tan\frac{\theta}{3}$

B.

$\frac{1}{3}\tan\theta$

C.

$\tan 3\theta$

D.

$\frac{1}{3}\tan 3\theta$

**Answer: B**

## Solution:

$$\frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right)=\tan^{-1}x$$

$$\Rightarrow \frac{1}{2}\sin^{-1}\left(\frac{3\cdot\frac{2\tan\theta}{1+\tan^2\theta}}{5+4\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)}\right)=\tan^{-1}x$$

$$\Rightarrow \sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)=2\tan^{-1}x$$

$$\Rightarrow \sin^{-1}\left(\frac{2\cdot\left(\frac{1}{3}\tan\theta\right)}{1+\left(\frac{\tan\theta}{3}\right)^2}\right)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\Rightarrow x=\frac{1}{3}\tan\theta$$

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# Question21

**If** $\operatorname{sech}^{-1}x=\log 2$ **and** $\operatorname{cosech}^{-1}y=-\log 3$**, then** $(x+y)=$

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**Options:**

A.

$\frac{1}{6}$

B.

$\frac{1}{20}$

C.

6

D.

20

**Answer: B**

## Solution:

$\operatorname{sech}^{-1} x = \log 2$

and $\operatorname{cosech}^{-1} y = -\log 3$

$\operatorname{sech}^{-1} x = \log 2$

$$\Rightarrow \quad \log\left(\frac{1+\sqrt{1-x^2}}{x}\right) = \log 2$$
$$\Rightarrow \quad \frac{1+\sqrt{1-x^2}}{x} = 2$$
$$\Rightarrow \quad 1+\sqrt{1-x^2} = 2x$$
$$\sqrt{1-x^2} = 2x-1$$

Squaring both sides, we get

$$\Rightarrow \quad 1-x^2 = 4x^2+1-4x$$
$$\Rightarrow \quad 5x^2-4x = 0$$
$$\Rightarrow \quad x(5x-4) = 0$$
$$x = \frac{4}{5}$$

$\because \operatorname{cosech}^{-1} y = -\log 3$

$$\Rightarrow \quad \log\left(\frac{1+\sqrt{1+y^2}}{y}\right) = \log\frac{1}{3}$$
$$\Rightarrow \quad \frac{1+\sqrt{1+y^2}}{y} = \frac{1}{3}$$
$$\Rightarrow \quad 1+\sqrt{1+y^2} = \frac{y}{3}$$
$$\Rightarrow \quad \sqrt{1+y^2} = \frac{y}{3}-1$$

Squaring both sides, we get

$$\Rightarrow \quad 1+y^2=\frac{y^2}{9}+1-\frac{2}{3}y$$

$$\Rightarrow \quad \frac{8y^2}{9}+\frac{2}{3}y=0$$

$$\Rightarrow \quad \frac{2}{3}y\left(\frac{4y}{3}+1\right)=0$$

$$\Rightarrow \quad y=\frac{-3}{4}$$

$$\Rightarrow \quad x+y=\frac{4}{5}-\frac{3}{4}=\frac{16-15}{20}$$

$$\Rightarrow \quad x+y=\frac{1}{20}$$

---

# Question22

**If $y=\tan^{-1}\left(\frac{x}{1+2x^2}\right)+\tan^{-1}\left(\frac{x}{1+6x^2}\right)$, then $\frac{dy}{dx}=$**

## AP EAPCET 2025 - 22nd May Evening Shift

**Options:**

A.

$\frac{4}{16x^2+1}-\frac{3}{9x^2+1}$

B.

$\frac{3}{9x^2+1}-\frac{1}{x^2+1}$

C.

$\frac{3}{9x^2+1}-\frac{2}{4x^2+1}$

D.

$\frac{1}{9x^2+1}-\frac{1}{x^2+1}$

**Answer: B**

## Solution:

$$y=\tan^{-1}\left(\frac{x}{1+2x^2}\right)+\tan^{-1}\left(\frac{x}{1+6x^2}\right)$$
$$\Rightarrow y=\tan^{-1}2x-\tan^{-1}x+\tan^{-1}3x$$
$$\Rightarrow y=\tan^{-1}3x-\tan^{-1}x$$
$$\Rightarrow \frac{dy}{dx}=\frac{1\times 3}{1+9x^2}-\frac{1}{1+x^2}$$
$$\Rightarrow \frac{dy}{dx}=\frac{3}{9x^2+1}-\frac{1}{x^2+1}$$

---

# Question23

**The range of the real valued function $f(x)=\cos^{-1}(-x)+\sin^{-1}(-x)+\operatorname{cosec}^{-1}(x)$ is**

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**Options:**

A.

$\{0, \frac{\pi}{2}\}$

B.

$[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

C.

$(0, \frac{\pi}{2})$

D.

$\{0, \pi\}$

**Answer: B**

## Solution:

$f(x)=\cos^{-1}(-x)+\sin^{-1}(-x)+\operatorname{cosec}^{-1}(x)]$
Domain $=(-1,1)$
$\Rightarrow \quad f(x)=\pi-\cos^{-1} x-\sin^{-1} x \quad+\operatorname{cosec}^{-1} x$
$\Rightarrow \quad f(x)=\pi-\frac{\pi}{2}+\operatorname{cosec}^{-1} x$
$\Rightarrow \quad f(x)=\frac{\pi}{2}+\operatorname{cosec}^{-1} x$

Domain of $\operatorname{cosec}^{-1} x$

$(-\infty,-1] \cup[1, \infty)$

The range of $\operatorname{cosec}^{-1} x$ is

$$\left[-\frac{\pi}{2}, 0\right) \cup\left(0, \frac{\pi}{2}\right]$$

$\because f(x)=\frac{\pi}{2}+\operatorname{cosec}^{-1} x$

$\therefore$ Range of

$$\begin{aligned} f(x) & =\left[\frac{\pi}{2}-\frac{\pi}{2}, \frac{\pi}{2}+0\right) \cup\left(0+\frac{\pi}{2}, \frac{\pi}{2}+\frac{\pi}{2}\right] \\ & =\left[0, \frac{\pi}{2}\right) \cup\left(\frac{\pi}{2}, \pi\right] \end{aligned}$$

---

# Question24

**The horizontal distance between a tower and a building is $10\sqrt{3}$ units. If the angle of depression of the foot of the building from the top of the tower is $60^{\circ}$ and the angle of elevation of the top of the building from the foot of the tower is $30^{\circ}$, then the sum of the heights of the tower and the building is**

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**Options:**

A.

60

B.

50

C.

40

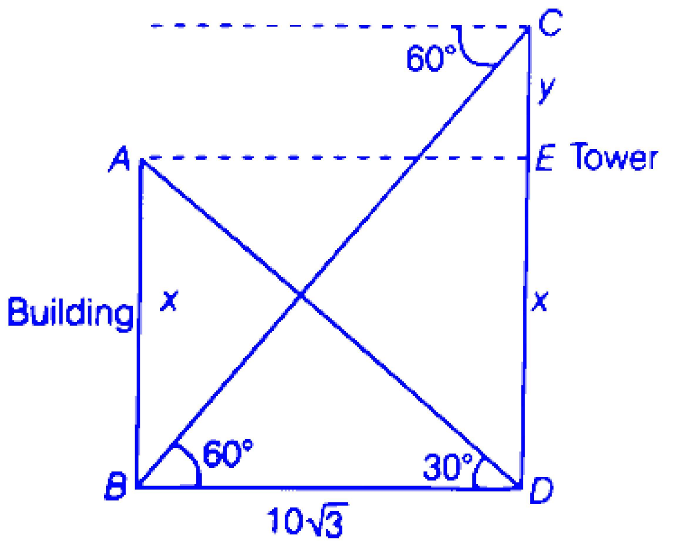
D.

30

**Answer: C**

## Solution:

In $\triangle ADB$,




$$\begin{aligned} \Rightarrow \quad \tan 30^{\circ} &= \frac{x}{10\sqrt{3}} \\ x &= 10 \qquad \ldots (i) \\ \triangle CDB, \tan 60^{\circ} &= \frac{x+y}{10\sqrt{3}} \\ x+y &= 30 \end{aligned}$$

$\therefore$ Sum of height $= 10 + 30 = 40$

---

# Question25

**If $x$ is a real number, then the number of solutions of $\tan^{-1}(\sqrt{x(x+1)}) + \sin^{-1}\left(\sqrt{x^2+x+1}\right) = \frac{\pi}{2}$ is**

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**Options:**

A.

1

B.

2

C.

3

D.

4

**Answer: B**

## Solution:

$$\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\left(\sqrt{x^2+x+1}\right)=\frac{\pi}{2}$$
$$\Rightarrow \tan^{-1}\sqrt{x(x+1)}=\cos^{-1}\left(\sqrt{x^2+x+1}\right)$$
$$\Rightarrow \tan^{-1}\sqrt{x(x+1)}=\tan^{-1}\sqrt{\frac{-x^2-x}{x^2+x+1}}$$
$$\Rightarrow \sqrt{x(x+1)}=\sqrt{\frac{-x^2-x}{x^2+x+1}}$$
$$\Rightarrow x(x+1)=\frac{-x(x+1)}{(x^2+x+1)}$$

So, $x(x+1)=0$ or $x^2+x+2=0$ (rejected)

$\Rightarrow x(x+1)=0 \Rightarrow x=0,-1$

$\because$ Only two real solution.

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# Question26

**If $y=\tanh^{-1}\sqrt{\frac{1-x}{1+x}}$, then $\frac{dy}{dx}=$**

### AP EAPCET 2025 - 21st May Evening Shift

**Options:**

A.

$-\frac{1}{2\sqrt{1-x^2}}$

B.

$\frac{-1}{2x\sqrt{1-x^2}}$

C.

$\frac{2}{1+x^2}$

D.

$\frac{1}{2x\sqrt{1+x^2}}$

**Answer: B**

## Solution:

$$
\begin{aligned}
& y=\tan h^{-1} \sqrt{\frac{1-x}{1+x}} \\
& \because \quad \frac{d}{d x} \tan h^{-1} x=\frac{1}{1-x^2} \\
& \therefore \quad \frac{d y}{d x}=\frac{1}{1-\frac{1-x}{1+x}} \times \frac{1}{2} \sqrt{\frac{1+x}{1-x}} \times \frac{(1+x)(-1)-(1-x)}{(1+x)^2} \\
& =\frac{1+x}{1+x-1+x} \times \frac{1}{2} \frac{\sqrt{1+x}}{\sqrt{1-x}} \times \frac{-2}{(1+x)^2} \\
& =-\frac{1+x}{2 x} \times \frac{1}{(1+x) \sqrt{(1+x)(1-x)}} \\
& =-\frac{1}{2 x \sqrt{1-x^2}}
\end{aligned}
$$

---

# Question27

$\tan^{-1} \frac{\sqrt{8-2\sqrt{15}}}{\sqrt{15}+1}+\tan^{-1} \frac{1}{\sqrt{5}}=$

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**Options:**

A.

$\frac{\pi}{6}$

B.

$\frac{\pi}{4}$

C.

$\frac{\pi}{3}$

D.

$\frac{\pi}{2}$

**Answer: A**

## Solution:

$$\tan^{-1}\frac{\sqrt{8-2\sqrt{15}}}{\sqrt{15}+1}+\tan^{-1}\frac{1}{\sqrt{5}}$$

$$=\tan^{-1}\left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{15}+1}\right)+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{15}+1}\times\frac{\sqrt{15}-1}{\sqrt{15}-1}\right)+\tan^{-1}\frac{1}{\sqrt{5}}$$

$$=\tan^{-1}\left(\frac{5\sqrt{3}-3\sqrt{5}-\sqrt{5}+\sqrt{3}}{14}\right)\quad+\tan^{-1}\frac{1}{\sqrt{5}}$$

$$=\tan^{-1}\left(\frac{6\sqrt{3}-4\sqrt{5}}{14}\right)+\tan^{-1}\frac{1}{\sqrt{5}}$$

$$=\tan^{-1}\left(\frac{3\sqrt{3}-2\sqrt{5}}{7}\right)+\tan^{-1}\frac{1}{\sqrt{5}}$$

$$=\tan^{-1}\left(\frac{\frac{3\sqrt{3}-2\sqrt{5}}{7}+\frac{1}{\sqrt{5}}}{1-\frac{3\sqrt{3}-2\sqrt{5}}{7}\times\frac{1}{\sqrt{5}}}\right)$$

$$=\tan^{-1}\left(\frac{3\sqrt{15}-10+7}{7\sqrt{5}-3\sqrt{3}+2\sqrt{5}}\right)$$

$$=\tan^{-1}\left(\frac{3\sqrt{15}-3}{9\sqrt{5}-3\sqrt{3}}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{15}-1}{3\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$=\tan^{-1}\left(\frac{3\sqrt{5}-\sqrt{3}}{3\sqrt{5}-\sqrt{3}}\times\frac{1}{\sqrt{3}}\right)$$

$$=\tan^{-1}\frac{1}{\sqrt{3}}=\frac{\pi}{6}$$

---

# Question28

**The derivative of** $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ **with respect to** $\sqrt{1-x^2}$ **at** $x=\frac{1}{2}$ **is**

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**Options:**

A.

-2

B.

1

C.

2

D.

4

**Answer: D**

## Solution:

$$\sec^{-1}\frac{1}{2x^2-1}=\cos^{-1}\left(2x^2-1\right)=2\cos^{-1}x$$
$$\because \quad f(x)=$$
$$\Rightarrow \quad f'(x)=-\frac{2\cos^{-1}x}{\sqrt{1-x^2}}$$
$$\Rightarrow \quad g(x)=\sqrt{1-x^2}$$
$$\Rightarrow \quad g'(x)=\frac{1}{2\sqrt{1-x^2}}\times(-2x)$$
$$=-\frac{x}{\sqrt{1-x^2}}$$
$$\Rightarrow \quad \frac{df}{dg}=\frac{-\frac{2}{\sqrt{1-x^2}}}{-\frac{x}{\sqrt{1-x^2}}}=\frac{2}{x}$$
$$\Rightarrow \left(\frac{df}{dg}\right)_{\text{at } x=\frac{1}{2}}=\frac{2}{\frac{1}{2}}=4$$

---

# Question29

**If** $0 < x < \frac{1}{2}$ **and** $\alpha = \sin^{-1} x + \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right)$**, then** $\tan\alpha + \cot\alpha$ **is equal to**

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**Options:**

A. $\frac{4}{\sqrt{3}}$

B. $4\sqrt{3}$

C. $\frac{4x}{1-x^2}$

D. $x\sqrt{1-x^2}$

**Answer: A**

## Solution:

We have, $0

$$\begin{aligned} \alpha &= \sin^{-1} x + \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right) \\ &= \sin^{-1} x + \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3}}{2}\sqrt{1-x^2}\right) \end{aligned}$$

Let $x = \sin\theta$

$$\begin{aligned} \alpha &= \sin^{-1}(\sin\theta) + \cos^{-1} \\ &= \theta + \cos^{-1}\left(\cos\left(\frac{\pi}{6} - \theta\right)\right) \\ &= \theta + \frac{\pi}{6} - \theta = \frac{\pi}{6} \end{aligned}$$

$$\begin{aligned} \tan\alpha + \cot\alpha &\Rightarrow \tan\frac{\pi}{6} + \cot\frac{\pi}{6} \\ &= \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{4}{\sqrt{3}} \end{aligned}$$

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# Question30

$\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ **is equal to**

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**Options:**

A. $\frac{26}{25}$

B. $\frac{25}{26}$

C. $\frac{50}{51}$

D. $\frac{52}{51}$

**Answer: A**

## Solution:

We have,

$$\cot\left[\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right] \qquad \ldots(i)$$

$$\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)$$

$$=\sum_{n=1}^{50}\tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$$

$$=\sum_{n=1}^{50}\left[\tan^{-1}(n+1)-\tan^{-1}n\right]$$

$$=\tan^{-1}51-\tan^{-1}1$$

$$=\tan^{-1}\left(\frac{51-1}{1+51\cdot 1}\right)=\tan^{-1}\left(\frac{50}{52}\right)$$

Now, from Eq. (i), we get

$$\Rightarrow \quad \cot\left(\tan^{-1}\left(\frac{50}{52}\right)\right)=\cot\left(\cot^{-1}\left(\frac{52}{50}\right)\right)$$

$$=\frac{52}{50}=\frac{26}{25}$$

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# Question31

## The value of $x$ such that $\sin\left(2\tan^{-1}\frac{3}{4}\right) = \cos\left(2\tan^{-1}x\right)$

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**Options:**

A. 7

B. $\frac{3}{7}$

C. $\frac{1}{7}$

D. $\frac{4}{7}$

**Answer: C**

## Solution:

To find the value of $x$, we need to equate:

$\sin\left(2\tan^{-1}\frac{3}{4}\right) = \cos\left(2\tan^{-1}x\right)$.

**Step-by-step Solution:**

**1. Left-hand side (LHS):**

Let $\theta = \tan^{-1}\frac{3}{4}$, which implies $\tan\theta = \frac{3}{4}$.

We have:

$\sin(2\theta) = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\times\frac{3}{4}}{1+\left(\frac{3}{4}\right)^2}$

Calculating further:

$= \frac{\frac{3}{2}}{1+\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{25}{16}} = \frac{24}{25}$.

**2. Right-hand side (RHS):**

Let $z = \tan^{-1}x$, hence $x = \tan z$.

Therefore:

$\cos(2z) = \frac{1-\tan^2 z}{1+\tan^2 z} = \frac{1-x^2}{1+x^2}$.

**3. Equating LHS to RHS:**

$\frac{24}{25} = \frac{1-x^2}{1+x^2}$.

Cross-multiplying gives:

$24 + 24x^2 = 25 - 25x^2$.

Simplifying:

$49x^2 = 1,$

which leads to:

$x^2 = \frac{1}{49} = \frac{1}{7^2},$

thus:

$x = \frac{1}{7}.$

----------------------------------------------------------------------------------------------------

# Question32

## The range of the real valued function $f(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right) + \cos^{-1}\left(\frac{2x}{1+x^2}\right)$ is

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**Options:**

A. $\left\{\frac{\pi}{2}\right\}$

B. $R$

C. $Q$

D. $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$

**Answer: A**

## Solution:

Given that

$f(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right) + \cos^{-1}\left(\frac{2x}{1+x^2}\right)$

Let $t = \frac{1+x^2}{2x}$ and $u = \frac{2x}{1+x^2}$

$\because \quad t = \frac{1}{u}$

$\therefore \quad t \cdot u \equiv 1$

We know that

$\sin^{-1}(a) + \cos^{-1}(a) = \frac{\pi}{2}, a \in [-1, 1]$

Thus, $\sin^{-1}(t) + \cos^{-1}(u) = \frac{\pi}{2}$

So, the range of the function

$f(x)=\sin^{-1}\left(\frac{1+x^2}{2x}\right)+\cos^{-1}\left(\frac{2x}{1+x^2}\right)$ is $\left\{\frac{\pi}{2}\right\}$

---

# Question33

## The real values of $x$ that satisfy the equation $\tan^{-1} x+\tan^{-1} 2x=\frac{\pi}{4}$ is

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**Options:**

A. $\frac{-3\pm\sqrt{17}}{4}$

B. $-1\pm\sqrt{3}$

C. $\sqrt{3}-1$

D. $\frac{\sqrt{17}-3}{4}$

**Answer: D**

## Solution:

Given $\tan^{-1} x+\tan^{-1} 2x=\frac{\pi}{4}$

We know that,

$\tan^{-1} x+\tan^{-1} y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy<1$

Now, $\tan^{-1} x+\tan^{-1} 2x=\frac{\pi}{4}$

$\Rightarrow \quad \tan^{-1}\left(\frac{x+2x}{1-x\cdot 2x}\right)=\frac{\pi}{4}$

$\Rightarrow \quad \tan^{-1}\left(\frac{3x}{1-2x^2}\right)=\frac{\pi}{4}$

$\Rightarrow \quad \frac{3x}{1-2x^2}=\tan\frac{\pi}{4}$

$\Rightarrow \quad \frac{3x}{1-2x^2}=1$

$\Rightarrow \quad \frac{3x}{1-2x^2}=1 \quad \left[\because \tan\frac{\pi}{4}=1\right]$

$\Rightarrow \quad 3x=1-2x^2$

$\Rightarrow \quad 2x^2+3x-1=0$

Using quadratic formula

$$x=\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Where, $a=2, b=3, c=-1$

$$x=\frac{-3 \pm \sqrt{9-4 \times 2 \times(-1)}}{4}$$

$$=\frac{-3 \pm \sqrt{9+8}}{4}$$

$$\Rightarrow \quad x=\frac{-3 \pm \sqrt{17}}{4}$$

Thus, $\quad x=\frac{-3+\sqrt{17}}{4}$, or $\frac{-3-\sqrt{17}}{4}$

$$For, \quad x=\frac{-3-\sqrt{17}}{4}$$

Since, $x \cdot 2x<1 \Rightarrow 2x^2<1$

$\Rightarrow 2\left(\frac{-3-\sqrt{17}}{4}\right)^2<1$, which is not true.

$\therefore \quad x=\frac{-3-\sqrt{17}}{4}$ is not possible

For $x=\left(\frac{-3+\sqrt{17}}{4}\right)$

Now, $\quad 2x^2<1$

$2\left(\frac{-3+\sqrt{17}}{4}\right)^2<1$

Hence, $x=\frac{\sqrt{17}-3}{4}$ is the only solution of the given equation.

---

# Question34

$2 \operatorname{coth}^{-1}(4)+\sec h^{-1}\left(\frac{3}{5}\right)=$

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**Options:**

A. $\log 5$

B. $2 \log 3$

C. $3 \log 2$

D. $\log \frac{5}{3}$

**Answer: A**

## Solution:

We have, $2 \coth^{-1}(4) + \operatorname{sech}^{-1}\left(\frac{3}{5}\right)$

We know that $\coth^{-1}(x) = \frac{1}{2} \log\left(\frac{x+1}{x-1}\right)$

$\operatorname{sech}^{-1}(x) = \log\left(\frac{1+\sqrt{1-x^2}}{x}\right), 0 < x \leq 1$

$$\therefore 2 \coth^{-1}(4) + \operatorname{sech}^{-1}\left(\frac{3}{5}\right)$$
$$= 2 \times \frac{1}{2} \log\left(\frac{4+1}{4-1}\right) + \log\left(\frac{1+\sqrt{1-\left(\frac{3}{5}\right)^2}}{\frac{3}{5}}\right)$$
$$= \log\left(\frac{5}{3}\right) + \log\left(\frac{1+\frac{1}{5}\sqrt{25-9}}{\frac{3}{5}}\right)$$
$$= [\log 5 - \log 3] + \log\left(\frac{5+\sqrt{16}}{3}\right)$$
$$= [\log 5 - \log 3] + \log\left(\frac{5+4}{3}\right)$$
$$= [\log 5 - \log 3) + \log\left(\frac{9}{3}\right)$$
$$= [\log 5 - \log 3] + \log 3$$
$$= \log 5$$

---

# Question35

**If $y = \sin^{-1} x$, then $\left(1 - x^2\right) y_2 - x y_1 =$**

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**Options:**

A. 0

B. 1

C. 2

D. $2y$

**Answer: A**

## Solution:

If $y = \sin^{-1} x$, then

$(1 - x^2) y_2 - x y_1 = \ ?$

We have, $\Rightarrow y = \sin^{-1} x$

On differentiating $y$ w.r.t. $x$, we get

$$\Rightarrow \quad \frac{dy}{dx} = y_1 = \frac{d}{dx}\left(\sin^{-1}(x)\right)$$
$$\Rightarrow \quad \frac{dy}{dx} = y_1 = \frac{1}{\sqrt{1 - x^2}} \qquad \ldots (i)$$

Again differentiate Eq. (i) w.r.t. $x$, we get

$$\Rightarrow y_2 = \frac{d^2 y}{dx^2} = \frac{d}{dx}\left(\frac{1}{\sqrt{1 - x^2}}\right)$$
$$\Rightarrow y_2 = \frac{x}{\sqrt{1 - x^2}\left(1 - x^2\right)}$$
$$\Rightarrow y_2\left(1 - x^2\right) = \frac{x}{\sqrt{1 - x^2}}$$

From Eq. (i), we get

$$y_2\left(1 - x^2\right) = x y_1$$
$$\Rightarrow \quad y_2\left(1 - x^2\right) - x y_1 = 0$$

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# Question36

### If $\cos^{-1} 2x + \cos^{-1} 3x = \frac{\pi}{3}$ and $4x^2 = \frac{a}{b}$, then $a + b$ is equal to

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**Options:**

A. 12

B. 11

C. 31

D. 10

**Answer: D**

## Solution:

Given, $\cos^{-1} 2x + \cos^{-1} 3x = \frac{\pi}{3}$

$$\cos^{-1}\left[(2x)(3x) - \sqrt{1-(2x)^2}\sqrt{1-(3x)^2}\right] = \frac{\pi}{3}$$
$$\Rightarrow 6x^2 - \sqrt{1-4x^2}\sqrt{1-9x^2} = \cos\frac{\pi}{3}$$
$$\Rightarrow 6x^2 - \frac{1}{2} = \sqrt{\left(1-4x^2\right)\left(1-9x^2\right)}$$

On squaring both sides, we get

$$36x^4 + \frac{1}{4} - 2\cdot\left(6x^2\right)\frac{1}{2} = 1 - 9x^2 - 4x^2 + 36x^4$$
$$\Rightarrow \quad \frac{1}{4} - 6x^2 = 1 - 13x^2 \Rightarrow 7x^2 = \frac{3}{4}$$
$$\Rightarrow \quad x^2 = \frac{3}{28} \Rightarrow x = \frac{\sqrt{3}}{2\sqrt{7}}$$
$$4x^2 = 4 \times \frac{3}{28} = \frac{3}{7} = \frac{a}{b} \quad \text{[given]}$$

Thus, $a + b = 3 + 7 = 10$

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# Question37

## If $\theta = \sec^{-1}(\cosh u)$, then $u =$

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### Options:

A. $\log_e\left(\cot\left(\frac{\theta}{2} - \frac{\pi}{4}\right)\right)$

B. $\log_e\left(\tan\left(\frac{\theta}{2} - \frac{\pi}{4}\right)\right)$

C. $\log_\theta\left(\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)\right)$

D. $\log_e\left(\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right)$

**Answer: D**

## Solution:

Given, $\theta = \sec^{-1}(\cos hu)$

$\Rightarrow \quad \sec\theta = \cos hu$

$\Rightarrow \quad u = \cos h^{-1}(\sec\theta)$

Since, $\cos h^{-1}(x) = \log\left(x + \sqrt{x^2 - 1}\right)$

$$\cos h^{-1}(\sec\theta) = \log\left(\sec\theta + \sqrt{\sec^2\theta - 1}\right)$$
$$= \log(\sec\theta + \tan\theta)$$
$$= \log\left(\frac{1 + \sin\theta}{\cos\theta}\right)$$
$$= \log\left(\frac{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}\right)$$
$$= \log\left(\frac{1 + \tan\frac{\theta}{2}}{1 - \tan\frac{\theta}{2}}\right)$$
$$= \log\left(\frac{\tan\frac{\pi}{4} + \tan\frac{\theta}{2}}{1 - \tan\frac{\pi}{4}\tan\frac{\theta}{2}}\right)$$
$$= \log\left(\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right)$$
$$\therefore \quad u = \log\left(\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right).$$

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# Question38

**If $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = k$, then $\sin^{-1}\left(\sqrt{\frac{k}{2}}\right) + \cos^{-1}\left(\frac{k}{3}\right) =$**

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**Options:**

A. $\frac{2\pi}{3}$

B. $\frac{3\pi}{4}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

**Answer: A**

## Solution:

We are given:

$\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}=k$

This can be rewritten as:

$\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\left[\cos(\pi-\frac{3\pi}{8})\right]^4+\left[\cos(\pi-\frac{\pi}{8})\right]^4=k$

Since $\cos(\pi-\theta)=-\cos(\theta)$, the expression simplifies to:

$\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{\pi}{8}=k$

Thus, we have:

$2\cos^4\frac{\pi}{8}+2\cos^4\frac{3\pi}{8}=k$

Next, using the values:

$\cos\frac{\pi}{8}=\sqrt{\frac{2+\sqrt{2}}{4}}\quad\text{and}\quad\cos\frac{3\pi}{8}=\sqrt{\frac{2-\sqrt{2}}{4}}$

Thus:

$2\left[\left(\frac{2+\sqrt{2}}{4}\right)^2+\left(\frac{2-\sqrt{2}}{4}\right)^2\right]=k$

Expanding these squares:

$\left(\frac{2+\sqrt{2}}{4}\right)^2=\frac{4+2\sqrt{2}+1}{16}=\frac{6+4\sqrt{2}}{16}$

$\left(\frac{2-\sqrt{2}}{4}\right)^2=\frac{4-2\sqrt{2}+1}{16}=\frac{6-4\sqrt{2}}{16}$

Adding these together:

$\frac{6+4\sqrt{2}}{16}+\frac{6-4\sqrt{2}}{16}=\frac{12}{16}=\frac{3}{4}$

Thus:

$k=2\times\frac{3}{4}=\frac{3}{2}$

Now, we compute:

$\sin^{-1}\left(\sqrt{\frac{k}{2}}\right)+\cos^{-1}\left(\frac{k}{3}\right)$

$=\sin^{-1}\left(\sqrt{\frac{3}{4}}\right)+\cos^{-1}\left(\frac{1}{2}\right)$

$=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)+\cos^{-1}\left(\frac{1}{2}\right)$

$=\frac{\pi}{3}+\frac{\pi}{3}=\frac{2\pi}{3}$

---

# Question39

$$4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}=$$

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**Options:**

A. $\frac{\pi}{12}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

**Answer: C**

# Solution:

We are given the expression:

$4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}$

Let's simplify this expression step-by-step:

**First, break the expression:**

$4\tan^{-1}\frac{1}{5}=2\left(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{5}\right)$

**Use the addition formula for arctan:**

$\tan^{-1}a+\tan^{-1}b=\tan^{-1}\left(\frac{a+b}{1-ab}\right)$

Applying this to $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{5}$:

$=\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{5}}{1-\frac{1}{5}\cdot\frac{1}{5}}\right)=\tan^{-1}\left(\frac{\frac{2}{5}}{\frac{24}{25}}\right)=\tan^{-1}\left(\frac{5}{12}\right)$

Therefore, $2\left(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{5}\right)=2\tan^{-1}\left(\frac{5}{12}\right)$

**Now incorporate the other terms:**

$2\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\left(\frac{1}{99}\right)-\tan^{-1}\left(\frac{1}{70}\right)$

**Simplify further using the arctan subtraction formula:**

$\tan^{-1}a-\tan^{-1}b=\tan^{-1}\left(\frac{a-b}{1+ab}\right)$

$\tan^{-1}\left(\frac{1}{99}\right)-\tan^{-1}\left(\frac{1}{70}\right)=\tan^{-1}\left(\frac{\frac{1}{99}-\frac{1}{70}}{1+\frac{1}{99}\cdot\frac{1}{70}}\right)$

$=\tan^{-1}\left(\frac{-29}{6931}\right)$

**Final simplification:**

$2\tan^{-1}\left(\frac{5}{12}\right)-\tan^{-1}\left(\frac{1}{239}\right)$

$\tan^{-1}\left(\frac{\frac{5}{6}}{1-\frac{25}{144}}\right)-\tan^{-1}\left(\frac{29}{6931}\right)$

**Combine using $\tan^{-1}\left(\frac{a-b}{1+ab}\right)$:**

$\tan^{-1}\left(\frac{120}{119}\right)-\tan^{-1}\left(\frac{1}{239}\right)$

**Arrive at the final form:**

Using the formula:

$\tan^{-1}\left(\frac{120\times239-119}{119\times239+120}\right)=\tan^{-1}(1)=\frac{\pi}{4}$

Thus, the simplified result of the given expression is $\frac{\pi}{4}$.

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# Question40

$$\cosh\left(\sinh^{-1}(\sqrt{8})+\cosh^{-1}5\right)=$$

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**Options:**

A. $\sqrt{6}+4\sqrt{2}$

B. $15+8\sqrt{3}$

C. $6\sqrt{6}+10\sqrt{2}$

D. $8-15\sqrt{3}$

**Answer: B**

## Solution:

To solve the expression $\cosh\left(\sinh^{-1}(\sqrt{8})+\cosh^{-1}5\right)$, follow the steps outlined below:

**Express the Inverse Hyperbolic Functions as Logarithms:**

$\sinh^{-1}(\sqrt{8})=\log(\sqrt{8}+\sqrt{8+1})$

Simplifying:

$=\log(\sqrt{8}+3)=\log(3+\sqrt{8})$

Similarly, for $\cosh^{-1}(5)$:

$\cosh^{-1}(5) = \log(5 + \sqrt{5^2 - 1})$

Simplifying:

$= \log(5 + \sqrt{24}) = \log(5 + 2\sqrt{6})$

**Combine the Logarithms:**

$\sinh^{-1}(\sqrt{8}) + \cosh^{-1}(5) = \log((3 + \sqrt{8}) \cdot (5 + 2\sqrt{6}))$

**Evaluate the Product Inside the Logarithm:**

Calculate:

$(3 + \sqrt{8})(5 + 2\sqrt{6}) = 3 \cdot 5 + 3 \cdot 2\sqrt{6} + \sqrt{8} \cdot 5 + \sqrt{8} \cdot 2\sqrt{6}$

$= 15 + 6\sqrt{6} + 5\sqrt{8} + 2\sqrt{48}$

Simplifying $\sqrt{8} = 2\sqrt{2}$ and $\sqrt{48} = 4\sqrt{3}$:

$= 15 + 6\sqrt{6} + 10\sqrt{2} + 8\sqrt{3}$

**Calculate the $\cosh(x)$ Value:**

Since $e^x = 15 + 10\sqrt{2} + 6\sqrt{6} + 8\sqrt{3}$, $\cosh(x)$ is given by:

$\cosh(x) = \frac{e^x + e^{-x}}{2}$

Simplify $\cosh(x)$:

$= \frac{1}{2}\left[(3 + 2\sqrt{2})(5 + 2\sqrt{6}) + (3 - 2\sqrt{2})(5 - 2\sqrt{6})\right]$

$= \frac{1}{2}[30 + 16\sqrt{3}] = 15 + 8\sqrt{3}$

Thus, the expression evaluates to $15 + 8\sqrt{3}$.

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# Question41

$\tan^{-1} 2 + \tan^{-1} 3 =$

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**Options:**

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{4}$

D. $\frac{5\pi}{4}$

**Answer: C**

## Solution:

To solve the expression $\tan^{-1} 2 + \tan^{-1} 3$, we can use the formula for the sum of inverse tangents:

$\tan^{-1} a + \tan^{-1} b = \tan^{-1}\left(\frac{a+b}{1-ab}\right)$

Applying this formula to our problem, we have:

$\tan^{-1} 2 + \tan^{-1} 3 = \tan^{-1}\left(\frac{2+3}{1-2\times 3}\right)$

Simplify the fraction:

$= \tan^{-1}\left(\frac{5}{1-6}\right) = \tan^{-1}(-1)$

We know that $\tan^{-1}(-1) = -\frac{\pi}{4}$. However, we typically express angles in the range 0 to $\pi$. Thus:

$\tan^{-1} 2 + \tan^{-1} 3 = \frac{3\pi}{4}$

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# Question42

## $\tan^{-1}(-2) - \tan^{-1}(3)$ is equal to

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**Options:**

A. $\frac{3\pi}{4}$

B. $\frac{-\pi}{6}$

C. $\frac{\pi}{6}$

D. $\frac{-3\pi}{4}$

**Answer: D**

## Solution:

$$\tan^{-1}(-2) - \tan^{-1}(3) = \tan^{-1}\left[\frac{(-2)-3}{1+(-2)(3)}\right]$$

$$= \tan^{-1}\left[\frac{-5}{-5}\right] = \tan^{-1} 1$$

From options, $\tan\frac{3\pi}{4} = -1$

$$\Rightarrow \quad \tan\left(\frac{-3\pi}{4}\right) = 1 \Rightarrow \tan^{-1}(1) = \frac{-3\pi}{4}$$

$$\therefore \tan^{-1}(-2) - \tan^{-1}(3) = \frac{-3\pi}{4}$$

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# Question43

**If $x = \sin\left(2\tan^{-1} 2\right), y = \cos\left(2\tan^{-1} 3\right)$ and $z = \sec\left(3\tan^{-1} 4\right)$, then**

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**Options:**

A. $x < y < z$

B. $y < z < x$

C. $z < x < y$

D. $z < y < x$

**Answer: D**

## Solution:

**Step 1: Analyze $x = \sin\left(2\tan^{-1} 2\right)$**

We start with the expression $x = \sin\left(2\tan^{-1} 2\right)$.

Let $\theta = \tan^{-1} 2$, which means that $\tan\theta = 2$. Now, use the double angle identity for sine:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

We need to find $\sin\theta$ and $\cos\theta$. Since $\tan\theta = \frac{\sin\theta}{\cos\theta} = 2$, we can use the Pythagorean identity to find $\sin\theta$ and $\cos\theta$:

$$\sin^2\theta + \cos^2\theta = 1$$

Let $\sin\theta = 2k$ and $\cos\theta = k$. From the identity:

$$(2k)^2 + k^2 = 1 \quad \Rightarrow \quad 4k^2 + k^2 = 1 \quad \Rightarrow \quad 5k^2 = 1 \quad \Rightarrow \quad k = \frac{1}{\sqrt{5}}$$

Thus, $\sin\theta = \frac{2}{\sqrt{5}}$ and $\cos\theta = \frac{1}{\sqrt{5}}$.

Now, using the double angle formula:

$$x = \sin(2\theta) = 2\left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{5}}\right) = \frac{4}{5}$$

**Step 2: Analyze $y = \cos\left(2\tan^{-1} 3\right)$**

Next, for $y = \cos\left(2\tan^{-1} 3\right)$, let $\alpha = \tan^{-1} 3$, so that $\tan\alpha = 3$.

Using the double angle identity for cosine:

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

We need to find $\sin\alpha$ and $\cos\alpha$. Since $\tan\alpha = 3$, we use the same method:

$$\sin^2\alpha + \cos^2\alpha = 1$$

Let $\sin\alpha = 3k$ and $\cos\alpha = k$, so:

$$(3k)^2 + k^2 = 1 \quad \Rightarrow \quad 9k^2 + k^2 = 1 \quad \Rightarrow \quad 10k^2 = 1 \quad \Rightarrow \quad k = \frac{1}{\sqrt{10}}$$

Thus, $\sin\alpha = \frac{3}{\sqrt{10}}$ and $\cos\alpha = \frac{1}{\sqrt{10}}$.

Now, applying the double angle identity for cosine:

$$y = \cos(2\alpha) = \left(\frac{1}{\sqrt{10}}\right)^2 - \left(\frac{3}{\sqrt{10}}\right)^2 = \frac{1}{10} - \frac{9}{10} = -\frac{8}{10} = -\frac{4}{5}$$

**Step 3: Analyze $z = \sec\left(3\tan^{-1} 4\right)$**

Lastly, for $z = \sec\left(3\tan^{-1} 4\right)$, let $\beta = \tan^{-1} 4$, so $\tan\beta = 4$.

We need the secant of $3\beta$, and we'll use the triple angle identity for secant:

$$\sec(3\beta) = \frac{1 + 2\tan^2\beta}{1 - \tan^2\beta}$$

For $\tan\beta = 4$, this gives:

$$z = \sec(3\beta) = \frac{1 + 2(4)^2}{1 - (4)^2} = \frac{1 + 2(16)}{1 - 16} = \frac{1 + 32}{-15} = \frac{33}{-15} = -\frac{11}{5}$$

**Step 4: Compare the values**

We have:

- $x = \frac{4}{5}$
- $y = -\frac{4}{5}$
- $z = -\frac{11}{5}$

Thus, the correct order is:

$$z < y < x$$

Therefore, the correct answer is D: $z < y < x$.

---

# Question44

$\frac{d}{dx}\left\{\sin^2\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)\right\}$ **is equal to**

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**Options:**

A. 0

B. $\frac{1}{2}$

C. $\frac{-1}{2}$

D. $-1$

**Answer: D**

## Solution:

Let $x = \cos 2\theta$

$$\Rightarrow \frac{d\theta}{dx} = -\frac{1}{\sin 2\theta}$$
$$\therefore \frac{d}{dx}\left\{\sin^{-1}\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)\right\} = \frac{d}{dx}\sin^2\theta = \sin 2\theta \frac{d\theta}{dx} = -1$$

---

# Question45

## If $y = \tan^{-1}\left\{\frac{ax-b}{bx+a}\right\}$, then $y'$ is equal to

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**Options:**

A. $\frac{1}{1+x^2} + \frac{a^2}{a^2+b^2}$

B. $\frac{1}{1+x^2}$

C. $\frac{1}{1+\left(\frac{ax-b}{bx+a}\right)^2}$

D. $\frac{bx+a}{1+(ax-b)^2}$

**Answer: B**

## Solution:

$$y' = \left[\frac{1}{1+\left(\frac{ax-b}{bx+a}\right)^2}\right]\frac{(bx+a)a-(ax-b)b}{(bx+a)^2}$$
$$= \left[\frac{1}{(a^2+b^2)(x^2+1)}\right](a^2+b^2) = \frac{1}{x^2+1}$$

---

# Question46

**For how many distinct values of $x$, the following $\sin\left[2\cos^{-1}\cot\left(2\tan^{-1}x\right)\right]=0$ holds?**

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**Options:**

A. 8

B. 2

C. 6

D. 4

**Answer: B**

## Solution:

$$\begin{aligned}
&\sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}\right]=0\\
\Rightarrow\quad & 2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}=\sin^{-1}(0)=0\\
\Rightarrow\quad & \cos^{-1}\left(\cot\left(2\tan^{-1}x\right)\right)=0\\
\Rightarrow\quad & \cot\left[\left(2\tan^{-1}x\right)\right]=\cos 0=1\\
\Rightarrow\quad & \cot\left[\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]=1\\
\Rightarrow\quad & \cot\left[\cot^{-1}\left(\frac{1-x^2}{2x}\right)\right]=1\\
\Rightarrow\quad & \frac{1-x^2}{2x}=1\\
\Rightarrow\quad & 1-x^2=2x\\
\Rightarrow\quad & x^2+2x-1=0
\end{aligned}$$

$$x=\frac{-2\pm\sqrt{4+4}}{2}=\frac{-2\pm2\sqrt{2}}{2}$$

$$x=-1\pm\sqrt{2}$$

Given equation holds for 2 values of $x$.

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# Question47

**If $\tan^{-1}\left[\frac{1}{1+1\cdot2}\right]+\tan^{-1}\left[\frac{1}{1+2\cdot3}\right]+\ldots+\tan^{-1}\left[\frac{1}{1+n(1+1)}\right]=\tan^{-1}[x]$, then $x$ is equal to**

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**Options:**

A. $\frac{1}{n+1}$

B. $\frac{n}{n+1}$

C. $\frac{1}{n+2}$

D. $\frac{n}{n+2}$

**Answer: D**

## Solution:

$$\begin{aligned}
&\tan^{-1}\left(\frac{1}{1+1\cdot2}\right)+\tan^{-1}\left(\frac{1}{1+2\cdot3}\right)+\ldots\\
&\quad+\tan^{-1}\left(\frac{1}{1+n(n+1)}\right)=\tan^{-1}(x)\\
\Rightarrow\quad&\tan^{-1}\left(\frac{2-1}{1+1\cdot2}\right)+\tan^{-1}\left(\frac{3-2}{1+2\cdot3}\right)+\ldots\\
&\quad+\tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right)=\tan^{-1}(x)\\
\Rightarrow\quad&\tan^{-1}(2)-\tan^{-1}(1)+\tan^{-1}(3)-\tan^{-1}(2)+\ldots\\
\Rightarrow\quad&\tan^{-1}(n+1)-\tan^{-1}(n)=\tan^{-1}(x)\\
\Rightarrow\quad&\tan^{-1}\left(\frac{n+1-1}{1+(n+1)\cdot1}\right)=\tan^{-1}(x)=\tan^{-1}(x)\\
\Rightarrow\quad&\tan^{-1}\left(\frac{n}{n+2}\right)=\tan^{-1}(x)\Rightarrow x=\frac{n}{n+2}
\end{aligned}$$

---

# Question48

**If $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, where $x^2\leq1$. Then, find $\frac{dy}{dx}$ is equal to**

**Options:**

A. $\frac{\pi}{4}+\frac{1}{2} \cos ^{-1}\left(x^2\right)$

B. $\frac{\pi}{4}-\frac{1}{2} \cos ^{-1}\left(x^2\right)$

C. $\frac{-x}{\sqrt{1-x^4}}$

D. $\frac{-2 x}{\sqrt{1-x^4}}$

**Answer: C**

## Solution:

$y=\tan ^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

Substituting $x^2=\cos 2 \theta$

$$
\begin{aligned}
\Rightarrow \quad & \theta=\frac{1}{2} \cos ^{-1}\left(x^2\right) \\
\therefore \quad & y=\tan ^{-1}\left(\frac{\sqrt{1+\cos 2 \theta}+\sqrt{1-\cos 2 \theta}}{\sqrt{1+\cos 2 \theta}-\sqrt{1-\cos 2 \theta}}\right) \\
& =\tan ^{-1}\left(\frac{\sqrt{2} \cos \theta+\sqrt{2} \sin \theta}{\sqrt{2} \cos \theta-\sqrt{2} \sin \theta}\right) \\
& =\tan ^{-1}\left(\frac{1+\tan \theta}{1-\tan \theta}\right) \\
& =\tan ^{-1}\left(\tan \left(\frac{\pi}{4}+\theta\right)\right) \Rightarrow y=\frac{\pi}{4}+\theta \\
\Rightarrow \quad & y=\frac{\pi}{4}+\frac{1}{2} \cos ^{-1}\left(x^2\right)
\end{aligned}
$$

$$
\therefore \quad \frac{d y}{d x}=\frac{1}{2} \cdot\left(\frac{-2 x}{\sqrt{1-x^4}}\right)=\frac{-x}{\sqrt{1-x^4}}
$$

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# Question49

**If $\int \frac{d x}{x\left(\sqrt{x^4-1}\right)}=\frac{1}{k} \sec ^{-1}\left(x^k\right)$, then the value of $k$ is equal to**

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**Options:**

A. 1

B. 2

C. 3

D. 4

**Answer: B**

## Solution:

$\frac{1}{k}\sec^{-1}\left(x^k\right)=\int\frac{dx}{x\sqrt{x^4-1}}=\frac{1}{2}\int\frac{2xdx}{x^2\sqrt{x^4-1}}$

Let

$$\begin{aligned}&\Rightarrow\quad 2xdx=dt\\&=\frac{1}{2}\int\frac{dt}{t\sqrt{t^2-1}}=\frac{1}{2}\sec^{-1}t\\&\Rightarrow\frac{1}{k}\sec^{-1}\left(x^k\right)=\frac{1}{2}\sec^{-1}\left(x^2\right)\Rightarrow k=2\end{aligned}$$

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